Example 4.3: Electromagnetic Theory and Axial Topology Actuators

General and explicit expressions for the electromagnetic torque and *emf* can be derived. We apply $\vec{F} = \oint_l i\, d\vec{l} \times \vec{B} = -i \oint_l \vec{B} \times d\vec{l}$, or in differential form $d\vec{F} = i\, d\vec{l} \times \vec{B}$, to find $\vec{T} = \vec{R} \times \vec{F}$. For a straight filament (conductor) in a uniform magnetic field, from $\vec{F} = i\vec{l} \times \vec{B}$ one has $\vec{F} = -i\vec{B} \times \oint_l d\vec{l}$.

Therefore, for $B(\theta_r) = B_{\max}\tanh(a\theta_r)$, we obtain

$$\vec{T} = \int_{r_{in}}^{r_{out}} i_a B_{\max} \tanh(a\theta_r)\, dr\, \vec{a}_z$$

where r_{out} and r_{in} are the outer and inner radii of the magnets as evident from images on Figure 4.22.

Recalling that $T_e = T_{eL} + T_{eR}$, $\theta_L(t) = \theta_{L0} - \theta_r(t)$, $\theta_R(t) = \theta_{R0} + \theta_r(t)$, one obtains T_e

The induced *emf* is $\mathscr{E} = \oint_l \vec{E}(t) \cdot d\vec{l} = -\frac{d}{dt}\int_s \vec{B}(t) \cdot d\vec{s} = -N\frac{d\Phi}{dt} = -\frac{d\psi}{dt}$. Hence,

$$\mathscr{E} = -N\frac{d\int_{r_{in}}^{r_{out}}\int_{\theta_{i\min}}^{\theta_{i\max}} B_{\max}\tanh(a\theta_r) r\, dr\, d\theta_i}{dt} = -\frac{r_{out}^2 - r_{in}^2}{2} N B_{\max}(\tanh a\theta_L + \tanh a\theta_R)\omega_r$$
$$= -\frac{r_{out}^2 - r_{in}^2}{2} N B_{\max}[\tanh a(\theta_{L0} - \theta_r) + \tanh a(\theta_{R0} + \theta_r)]\omega_r.$$

As covered, the derived expressions for the electromagnetic torque and *emf* should be substituted in the Kirchhoff's and Newton's laws. ■

4.4 Translation Permanent-Magnet Electromechanical Motion Devices

Various rotational permanent-magnet devices and DC electric machines were covered in Sections 4.1 through 4.3. The translational (linear) devices have been designed and utilized in many applications. For example, speakers and microphones are actuators (motors) and generators, respectively. Alexander Graham Bell received a patent on the electromagnetic loudspeaker in 1876, and Nicola Tesla demonstrated other designs in 1881. The so-called moving *coil* speaker was proposed and demonstrated by Oliver Lodge in 1898. In these early designs, the stationary magnetic field was established by the *field coils* (electromagnet). The performance of speakers and microphones was significantly enhanced by using permanent magnets to establish a stationary magnetic field. The images of speakers with radially magnetized permanent magnets are illustrated in Figure 4.30.

FIGURE 4.30
Speakers as limited-displacement permanent-magnet DC actuators.

A lightweight cone or dome shape diaphragm is connected to a rigid frame using a flexible suspension. A variety of different materials are used, and the most common are paper, plastic, and coated and composite materials. An N-turn winding (*voice coil*) is under the stationary magnetic field established by radially magnetized permanent magnet or magnets as illustrated in Figure 4.30. To displace a diaphragm, one applies the voltage to the winding, and the electromagnetic force is produced. The suspension system maintains the coil centered within the gap and provides a restoring force to make the speaker cone return to a neutral position (equilibrium) if voltage is not applied. Insulated copper and silver wire is used to fabricate a *voice coil* within a circular, rectangular, or hexagonal cross section. The coil is oriented coaxially inside the gap. Ceramic, ferrite, alnico, and rear-earth (samarium cobalt, neodymium iron boron, and other) permanent magnet are used.

The analysis, design, and optimization tasks can be performed by applying the results reported. In particular, the *emf* and electromagnetic force are $emf = \oint_l \vec{E} \cdot d\vec{l} = \oint_l \left(\vec{v} \times \vec{B}\right) \cdot d\vec{l} - \oint_s \frac{\partial \vec{B}}{\partial t} d\vec{s}$ and $\vec{F} = \oint_l i\, d\vec{l} \times \vec{B} = -i \oint_l \vec{B} \times d\vec{l}$. Applying the Kirchhoff's voltage law and Newtonian mechanics, one obtains the resulting equation of motions. These equations are significantly affected by the speaker and microphone design, magnetic system, geometry, kinematics, etc. Piezoelectric, electrostatic, thermal, and other speakers are also used in low-performance application particularly if the specifications on the low cost and small size are imposed.

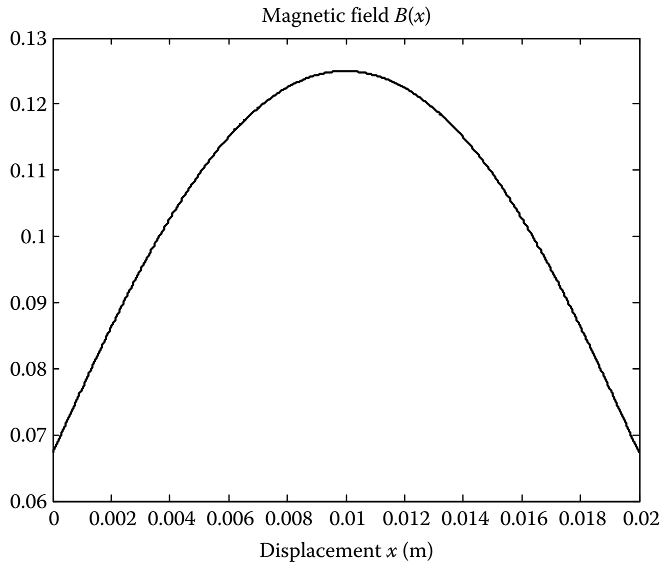

FIGURE 4.31
Plots of $B(x)$ with $x_0 = 0.01$ m, $l_{\min} \le x \le l_{\max}$, $l_{\min} = 0$, $l_{\max} = 0.02$ m, $B_{\max} = 0.25$ T, and $a = 10{,}000$.

Example 4.4:

Consider a speaker with a radially magnetized ring magnet which is reported in Figure 4.30. The distribution of $B(\mathbf{r})$, as viewed from the coil, significantly affects the overall performance and capabilities. Magnet is magnetized to ensure the desired $B(\mathbf{r})$, and one may approximate $B(\mathbf{r})$ using various continuous differentiable functions. For example, for one-dimensional field, trigonometric, exponential, sigmoid $\left(B = B_{\max}\frac{1}{1+e^{-ax}},\ B = B_{\max}\frac{1}{1+e^{-a|x|}},\ B = B_{\max}\frac{1}{1+e^{-ax^2}},\ B = B_{\max}\frac{1}{1+e^{-ax^3}}\right)$, $a > 0$, and other functions can be used depending on the magnet magnetization, relative displacement of coils with respect to magnet, magnet and coil geometry and orientation, magnet–coil separation, etc. The plot for the axial field $B(x)$ is documented in Figure 4.31. At equilibrium when $\Sigma F = 0$, the displacement is denoted as x_0. Considering the motion in the x-direction, which is constrained within $l_{\min} \le x \le l_{\max}$, the distribution of $B(x)$, as viewed from the coils with a high (width) $l_h = (l_{\max} - l_{\min})$, is of a particular importance. The cone displacement x is usually less than l_h, e.g., $x < l_h$. The MATLAB statement used to perform calculations and plotting is

```
lmin=0; lmax=0.02; x=0:(lmax-lmin)/1000:lmax;
Bmax=0.25; a=10000; B=Bmax.*(1-1./(1+exp(-a*(x-(lmax-lmin)/2).^2)));
plot(x,B,'LineWidth',2);
xlabel('Displacement x [m]','FontSize',14);
title('Magnetic Field B(x)','FontSize',14);
```

■

Using Kirchhoff's voltage law and Newtonian dynamics, one obtains

$$\frac{di_a}{dt} = \frac{1}{L_a}(-r_a i_a - emf + u_a),$$
$$\frac{dv}{dt} = \frac{1}{m}\left(i_a \int B(x)dl - F_{\text{air}} - F_{\text{elastic}} - F_{\xi}\right),$$
$$\frac{dx}{dt} = v,$$

where F_{air} is the air friction force, F_{elastic} is the elastic restoring force, and F_{ξ} is the disturbance force which is of a stochastic origin.

The electromagnetics of various electromechanical motion devices is covered. By using B ($\mathbf{r}$) and coil geometry, the *emf* and F_e are straightforwardly derived. The expressions for the air friction, elastic restoring forces, and stochastic forces of the mechanical origin can be obtained. Only for a preliminary design, one may apply the following approximations $F_{\text{elastic}} = k_{\text{elastic}} x$ and $F_{\text{air}} = k_{\text{air}} v$, where k_{elastic} and k_{air} are the constants. Coherent expressions for F_{elastic} and F_{air} should be used.

References

1. Chapman, S. J., *Electric Machinery Fundamentals*, McGraw-Hill, New York, 1999.
2. Fitzgerald, A. E., Kingsley, C., and Umans, S. D., *Electric Machinery*, McGraw-Hill, New York, 1990.
3. Krause, P. C. and Wasynczuk, O., *Electromechanical Motion Devices*, McGraw-Hill, New York, 1989.
4. Krause, P. C., Wasynczuk, O., and Sudhoff, S. D., *Analysis of Electric Machinery*, IEEE Press, New York, 1995.
5. Lyshevski, S. E., *Electromechanical Systems, Electric Machines, and Applied Mechatronics*, CRC Press, Boca Raton, FL, 1999.
6. Lyshevski, S. E., *Electromechanical Systems and Devices*, CRC Press, Boca Raton, FL, 2008.
7. Slemon, G. R., *Electric Machines and Drives*, Addison-Wesley Publishing Company, Reading, MA, 1992.
8. White, D. C. and Woodson, H. H., *Electromechanical Energy Conversion*, Wiley, New York, 1959.