ACCURATE EVALUATION OF BACKSCATTERING
BY 90° DIHEDRAL CORNERS

P. Corona, A. De Bonitatibus, and G. Ferrara
*Istituto di Teoria e Tecnica delle Onde Elettromagnetiche*
*Istituto Universitario Navale*
*via Acton 38, 80133 Napoli, Italy*

C. Gennarelli, *Dipartimento di Ingegneria dell'Informazione*
*e Matematica Applicata, Università di Salerno*
*84084 Fisciano (Salerno), Italy*

## ABSTRACT

A very accurate mathematical model for the evaluation of the backscattering by a perfectly conducting 90° dihedral corner has been developed. It has been obtained by adding the Physical Theory of Diffraction (PTD) correction term to the Improved Physical Optics (IPO) model, which takes into account also the lighting of each face by the rays diffracted from the edge of the other one. The agreement of such a model with the experimental results has been found very good.

## 1. INTRODUCTION

The 90° dihedral corner reflector (Fig.1) is formed by two flat metal plates placed at a right angle each other. It exhibits the important feature that a ray, which enters the corner normally impinging with respect to the dihedral wedge, is reflected from both the faces and returns in the direction from which it came. As a consequence, it yields a large Radar Cross Section (RCS) over a wide angular range in a plane normal to its wedge. Due to this last peculiarity, it can be used as radar enhancement device [1] and as a practical reference target for RCS measurements [2]÷[4].

On the other hand, it is a very interesting target also from a theoretical point of view. Ray-optics terms of various order are always simultaneously present in every backscattering direction, thus allowing to test their quantitative relevance.

The interest on the dihedral corner is confirmed by the publication, in recent years, of several papers dealing with the backscattering by perfectly conducting [2]÷[12], as well as loaded [13]÷[16], dihedral corners.

In [5], to determine the backscattered field in a plane normal to the dihedral wedge, the Physical Optics (PO) alone is used, whereas in [6] is employed also the Physical Theory of Diffraction (PTD) and multiple reflections (up to the triple) are included. A closed form PTD solution for the electromagnetic scattering by strips is derived in [7] and applied to the

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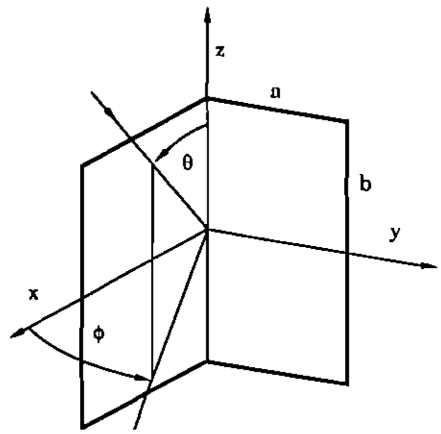


FIGURE 1. The dihedral corner reflector.

evaluation of the field backscattered by the 90° dihedral. In [8], to investigate the consequences of nonorthogonality on the scattering properties of dihedral corner reflectors, a PO model, which includes also triply reflected contributions is developed. In [9] a Geometrical Theory of Diffraction (GTD) model, including higher order reflections and diffractions, is adopted. The scattering of a plane wave normally impinging with respect to the wedge of an arbitrary interior angle dihedral reflector is evaluated by using the Spectral Domain Technique (SDT) in [10]. Whereas in [11] the Discrete Fourier Transform Method (DFTM) is employed to compute the RCS of orthogonal or nonorthogonal dihedral reflectors.

In the authors' paper [4], by developing a PO model which allows to handle the corner also in the case of non-normal incidence, the radiation characteristics of the perfectly conducting right angled dihedral corner have been examined in detail, showing that with the proper choice of the scanning plane, it can be conveniently used as a reference target in experimental determinations of RCS. Furthermore, to evaluate accurately the RCS of the corner in the (less general but more interesting) case of normal incidence, it has been developed an unusual PO model, which takes into account the lighting of each face by the rays diffracted from the edge of the other one. Such an Improved Physical Optics (IPO) model has been successively extended to the case of loaded corner [13].

At last, the measurement of the complex permittivity of lossy dielectrics employing a 90° dihedral corner, with the faces loaded by a layer of the dielectric under test, has been proposed by the authors [14]÷[16].

In this work an even more accurate model for the backscattering by a perfectly conducting 90° dihedral corner is presented. Such a model has been obtained by adding to the IPO model the PTD contribution [17]÷[19] and allows to predict very accurately the monostatic RCS of the corner in a plane normal to its wedge.

## 2 . THE MATHEMATICAL MODEL

A point of the corner is specified (Fig.1) by the cartesian coordinates (x,y,z), while the observation point by the spherical coordinates (r,θ,φ).

As it is well known, the RCS is defined as:

$$\sigma = \lim_{r\to\infty} 4\pi r^2 \frac{|\underline{E}^s|^2}{|\underline{E}^i|^2} \tag{1}$$

wherein $\underline{E}^i$ is the incident electric field on the target and $\underline{E}^s$ is the field backscattered at the observation point.

In the PO approach the spherical components of $\underline{E}^s$ are given by

$$\underline{E}^s = \frac{-jkZ_0}{4\pi} \frac{e^{-jkr}}{r} \iint_S \underline{\underline{M}}_1 \cdot \underline{J}_s \, e^{jk\underline{\rho}\cdot\hat{r}} \, dS \tag{2}$$

where k is the wavenumber, $Z_0$ is the free-space impedance, $\underline{\rho}$ is the vector from the origin to the integration point, $\underline{J}_s$ is the PO current distribution (whose components are expressed in the cartesian coordinates system) and, being $\theta = \pi/2$,

$$\underline{\underline{M}}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \tag{3}$$

As previously referred, the IPO model takes into account the lighting of the corner faces by the rays directly impinging, by the rays reflected and also by the rays diffracted from the edges.

The resulting expressions [4] for the backscattered field, considering, due to the symmetry, only the angular range $0 \le \phi \le \pi/4$, are for the normal polarization [(1)]:

$$\underline{E}_\perp^{IPO} = \frac{jk}{4\pi} 2abE_0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \frac{e^{-jkr}}{r} \Bigg\{ \sin\phi \, \frac{\sin(ka\cos\phi)}{ka\cos\phi} \, e^{jka\cos\phi} + \cos\phi \cdot$$

$$\cdot \frac{\sin(ka\sin\phi)}{ka\sin\phi} \, e^{jka\sin\phi} - 2\sin\phi + \frac{e^{-j\pi/4}}{a\sqrt{2\pi k}} \Bigg[ e^{jka\sin\phi} \cdot$$

$$\cdot \int_0^a \cos\gamma \; d_s(\Phi_2,\Phi_2') \, \frac{e^{-jkd_2}}{\sqrt{d_2}} \, e^{jkx\cos\phi} \, dx + e^{jka\cos\phi} \, .$$

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(1) The normal polarization corresponds to an electric field normal to the incidence plane.

$$\cdot \int_0^a \cos\Phi_1 \; d_s(\Phi_1,\Phi_1') \; \frac{e^{-jkd_1}}{\sqrt{d_1}} \; e^{jky\sin\phi} \, dy \Bigg]\Bigg]\Bigg\} \tag{4}$$

and for the parallel one:

$$\underline{E}_{\parallel}^{IPO} = \frac{jk}{4\pi} \, 2abE_0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \frac{e^{-jkr}}{r} \Bigg\{ \sin\phi \, \frac{\sin(ka\cos\phi)}{ka\cos\phi} \, e^{jka\cos\phi} + \cos\phi \, \cdot$$

$$\cdot \, \frac{\sin(ka\sin\phi)}{ka\sin\phi} \, e^{jka\sin\phi} + 2\sin\phi + \frac{e^{-j\pi/4}}{a\sqrt{2\pi k}} \Bigg[ \sin\phi \, e^{jka\sin\phi} \, \cdot$$

$$\cdot \int_0^a d_h(\Phi_2,\Phi_2') \, \frac{e^{-jkd_2}}{\sqrt{d_2}} \, e^{jkx\cos\phi} \, dx + \cos\phi \, e^{jka\cos\phi} \, \cdot$$

$$\cdot \int_0^a d_h(\Phi_1,\Phi_1') \, \frac{e^{-jkd_1}}{\sqrt{d_1}} \, e^{jky\sin\phi} \, dy \Bigg]\Bigg\} \tag{5}$$

where

$$d_{s,h}(\Phi,\Phi') = -\frac{1}{2} \left\{ \frac{F[kLa(\Phi-\Phi')]}{\cos[(\Phi-\Phi')/2]} \mp \frac{F[kLa(\Phi+\Phi')]}{\cos[(\Phi+\Phi')/2]} \right\} \tag{6}$$

are, except for the $e^{-j\pi/4}/\sqrt{2\pi k}$ factor explicitly pointed out, the Kouyoumjian diffraction coefficients for a perfectly conducting half plane [20], and the other symbols (Fig. 2) are reported in Table 1.

In (6)

$$F(x) = 2j\sqrt{x} \; e^{jx} \int_{\sqrt{x}}^{\infty} e^{-j\tau^2} \, d\tau \tag{7}$$

is the Kouyoumjian's transition function, $a(x) = 2\cos^2(x/2)$ and the distance parameter L is, in this case, the distance $d_1$ or $d_2$.

It is worthy to note that such a PO model is unusual since it uses GTD instead of GO to get the current distribution.

**TABLE 1.**

| | | | |
|---|---|---|---|
| $\Phi_1' = \pi - \phi$ | $\Phi_1 = tg^{-1}(y/a)$ | $\Phi_2' = 3\pi/2 - \phi$ | $\Phi_2 = 2\pi - \gamma$ |
| $\gamma = tg^{-1}(x/a)$ | $d_1 = a/\cos\Phi_1$ | $d_2 = a/\cos\gamma$ | |

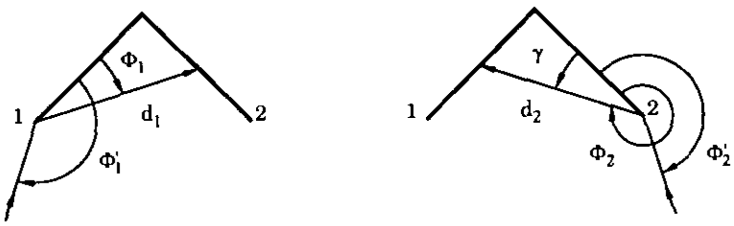



FIGURE 2. Relevant to diffracted fields contributions.

A further improvement to the model can be achieved by taking into account the rays diffracted from the faces edges parallel to the corner wedge and coming back directly to the observation point. This can be done by adding to the IPO model the PTD correction term.

Accordingly, it results:

$$\underline{E}_\perp = \underline{E}_\perp^{IPO} + \frac{jk}{4\pi}\, 2abE_0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \frac{e^{-jkr}}{r} \left\{ + \frac{j}{ka} \left[ e^{j2ka\cos\phi}\, d_s^u(\Phi_1^u, \Phi_1'^u) + e^{j2ka\sin\phi}\, d_s^u(\Phi_2^u, \Phi_2'^u) \right] \right\} \tag{8}$$

$$\underline{E}_{\shortparallel} = \underline{E}_{\shortparallel}^{IPO} + \frac{jk}{4\pi}\, 2abE_0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \frac{e^{-jkr}}{r} \left\{ - \frac{j}{ka} \left[ e^{j2ka\cos\phi}\, d_h^u(\Phi_1^u, \Phi_1'^u) + e^{j2ka\sin\phi}\, d_h^u(\Phi_2^u, \Phi_2'^u) \right] \right\} \tag{9}$$

wherein

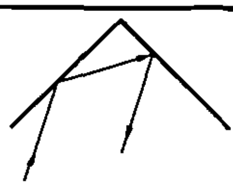
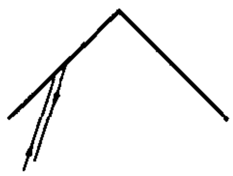
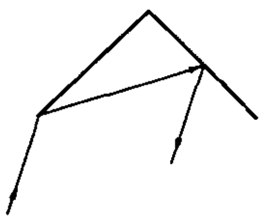
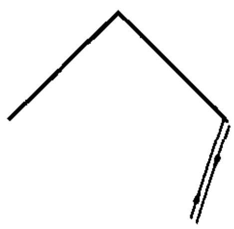
$$d^{u}_{s,h}(\Phi',\Phi') = -\frac{1}{2}\left[1 \mp \frac{1-\sin\Phi'}{\cos\Phi'}\right] \tag{10}$$

are the Ufimtsev diffraction coefficients [17], [18], and

$$\Phi^{u}_{1} = \pi - \phi \qquad \Phi^{u}_{2} = \pi/2 + \phi \tag{11}$$

As it can be seen (Table 2), the solution is the sum of four contributions.

**TABLE 2.**

| | | |
|---|---|---|
| i) |  | Backscattering from the faces lighted by the reflected rays. |
| ii) |  | Backscattering due to the direct lighting by the the *impinging wave.* |
| iii) |  | Backscattering due to the lighting of each face by the rays diffracted from the edge of the other one. |
| iv) |  | Backscattering due to the rays diffracted from the edges (PTD correction term). |

With reference to (4) such contributions are:

i) $$-\frac{jk}{4\pi}\, 2abE_0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \frac{e^{-jkr}}{r}\, 2\sin\phi$$

ii)
$$\frac{jk}{4\pi}\, 2abE_0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \frac{e^{-jkr}}{r} \left\{ \sin\phi\, \frac{\sin(ka\cos\phi)}{ka\cos\phi}\, e^{jka\cos\phi} + \right.$$
$$\left. + \cos\phi\, \frac{\sin(ka\sin\phi)}{ka\sin\phi}\, e^{jka\sin\phi} \right\}$$

iii)
$$\frac{jk}{4\pi}\, 2abE_0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \frac{e^{-jkr}}{r} \left\{ \left[ \int_0^a \cos\gamma\, d_s(\Phi_2,\Phi_2')\, \frac{e^{-jkd_2}}{\sqrt{d_2}}\, e^{jkx\cos\phi}\, dx \; + \right.\right.$$
$$\left.\left. + e^{jka\cos\phi} \int_0^a \cos\Phi_1\, d_s(\Phi_1,\Phi_1')\, \frac{e^{-jkd_1}}{\sqrt{d_1}}\, e^{jky\sin\phi}\, dy \right]\right\}$$

It is worthy to note that, if the RCS is evaluated, the term corresponding to i) coincides with the RCS value $\sigma = 16\,\pi\, a^2\, b^2 \sin^2\phi\, /\, \lambda^2$ obtained by Robertson [1] by considering the corner "effective area".

The first order contribution (i) is the first order term of the "interaction" scattering and the second order contribution (ii) is the first order term of the "direct" scattering, while the contribution (iii) is the second order term of the "interaction" scattering and (iv) is the second order term of the "direct" scattering. As a consequence, it may be expected that generally (iii) is the most meaningfull correction to the PO model. It must be stressed that in this case (90° dihedral corner) no triple reflection occurs.

Higher order contributions can be neglected from a practical point of view since they are quantitatively small. This is justified by evaluations performed and it is clearly confirmed by the very good agreement of the present model with the experimental results.

## 3. NUMERICAL AND EXPERIMENTAL RESULTS

Many numerical and experimental tests have been carried out in order to evaluate the accuracy of the complete model and of the partial ones. Two representative cases, which refer to the normal polarization and to right angled dihedral corners with $a/\lambda = 3.33$ and $a/\lambda = 5.607$, are reported in the following. In the former case the measurements have been performed by the authors in the anechoic chamber of the Istituto Universitario Navale at Naples. In the latter it has been made reference, for the sake of general availability, to an experimental pattern available in literature [21] (reported under kindly permission of the author and of the first publishers [22]).

Figs. 3÷6 refer to the dihedral corner with $a/\lambda = 3.33$ and Figs. 7÷10 to that

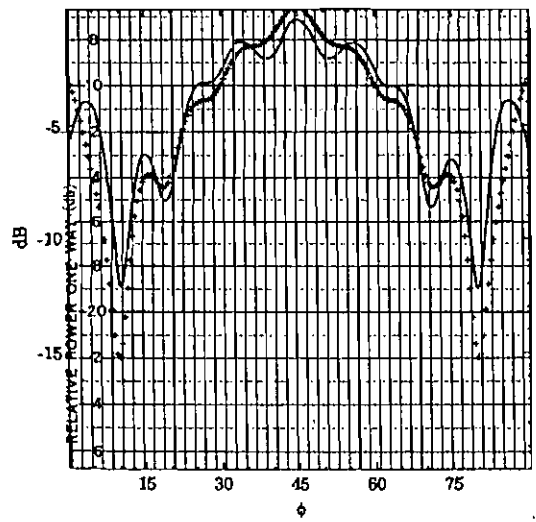



FIGURE 3. Field backscattered by a corner ($a/\lambda = 3.33$) measured ( —— ) and evaluated by means of PO (+++).

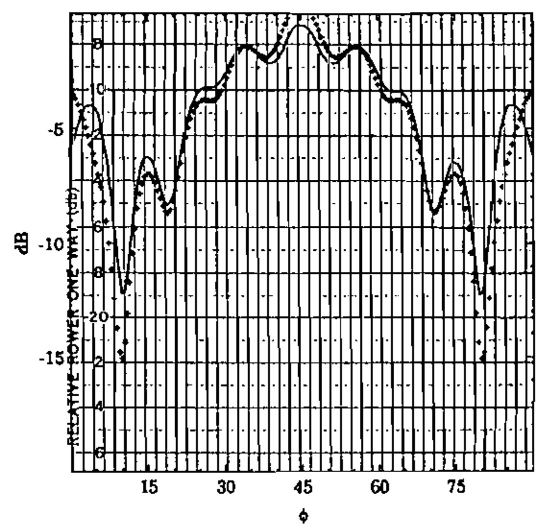



FIGURE 4. Field backscattered by a corner ($a/\lambda = 3.33$) measured ( —— ) and evaluated by means of PO + PTD (+++).

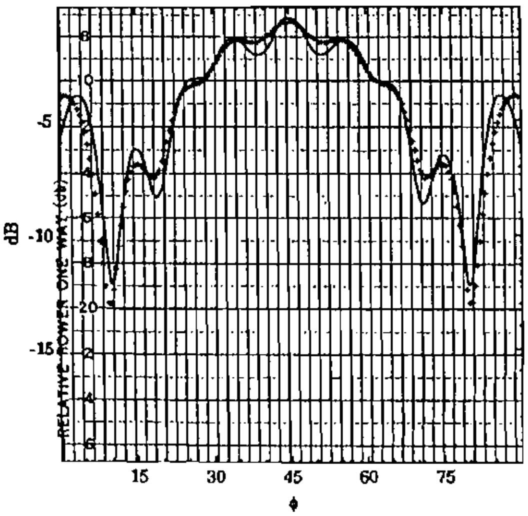



FIGURE 5. Field backscattered by a corner ($a/\lambda = 3.33$) measured ( — ) and evaluated by means of IPO (+++).

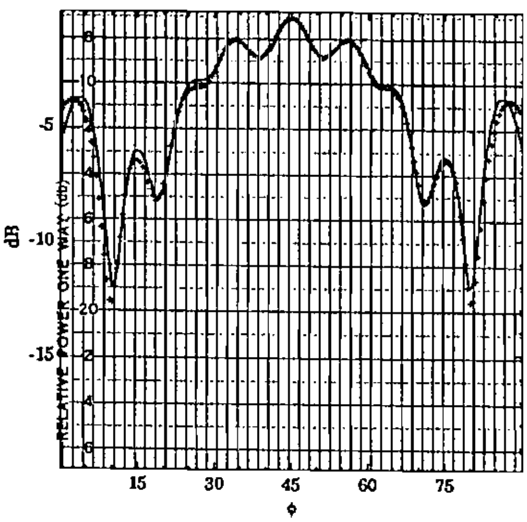



FIGURE 6. Field backscattered by a corner ($a/\lambda = 3.33$) measured ( — ) and evaluated by means of IPO + PTD (+++).

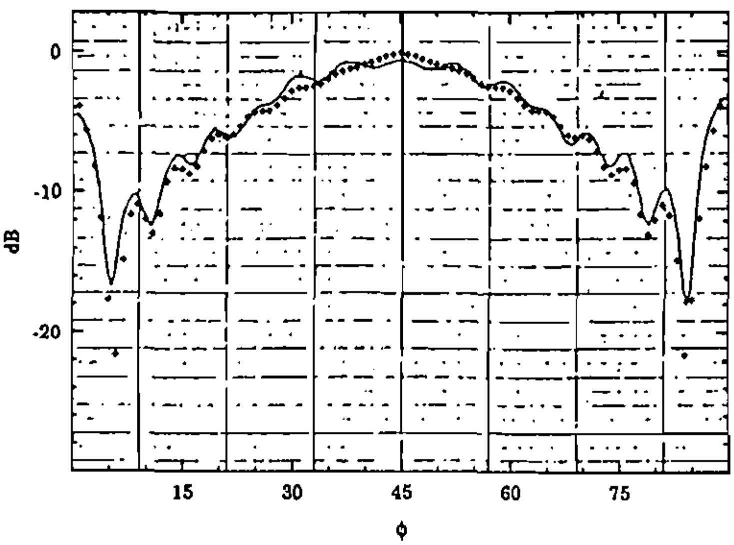



FIGURE 7. Field backscattered by a corner ($a/\lambda = 5.607$) measured ( — ) and evaluated by means of PO (+++).

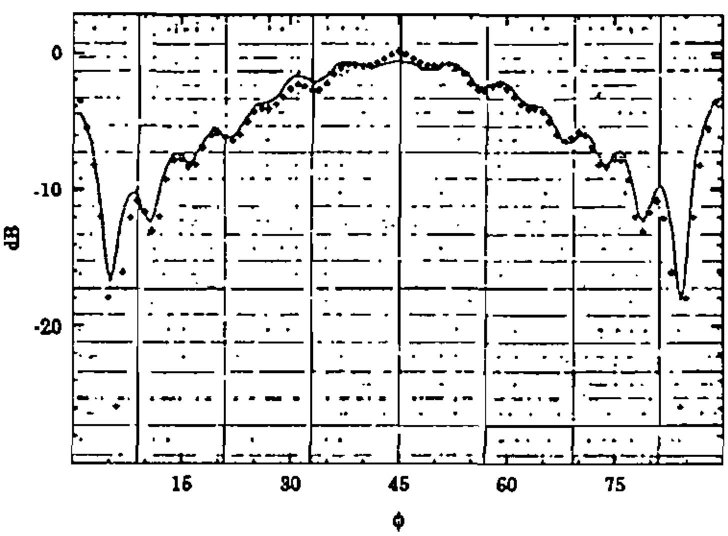



FIGURE 8. Field backscattered by a corner ($a/\lambda = 5.607$) measured ( — ) and evaluated by means of PO + PTD (+++).

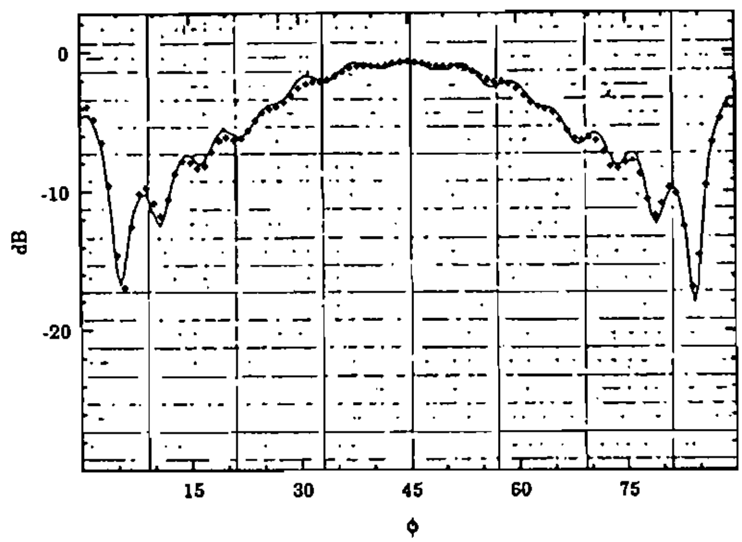



FIGURE 9. Field backscattered by a corner ($a/\lambda$ = 5.607) measured ( — ) and evaluated by means of IPO (+++).

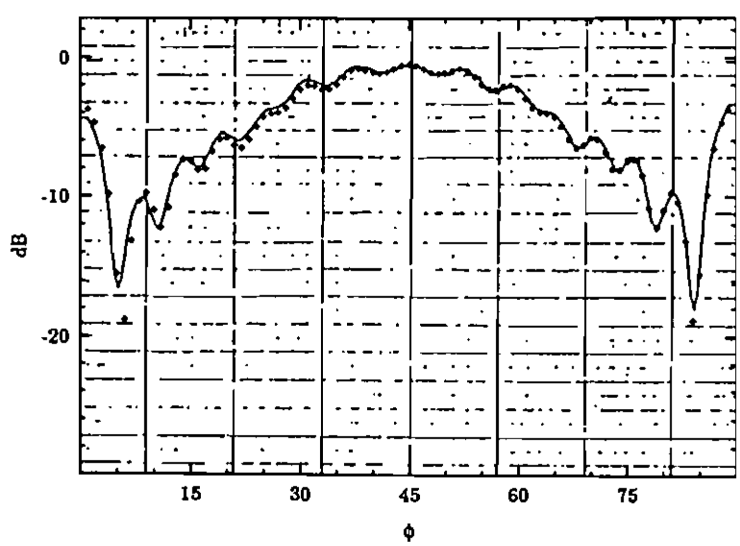



FIGURE 10. Field backscattered by a corner ($a/\lambda$ = 5.607) measured ( —— ) and evaluated by means of IPO + PTD (+++).

with $a/\lambda$ = 5.607. The graphs show the backscattered field (in dB), computed (crosses) with the different models and measured (solid line), versus the azimuthal angle $\phi$.

In both the cases, the complete model here developed fits almost perfectly the experimental results. Consequently, it can be considered as reference and the accuracy of the partial models can be estimated by computing the corresponding mean square errors.

The results relevant to the complete angular range $0° \leq \phi \leq 90°$ and to the partial one $40° \leq \phi \leq 50°$ are reported in Table 3 for the corner with $a/\lambda$ = 3.33 and in Table 4 for that with $a/\lambda$ = 5.607.

As a conclusion, it can be stated that the correction term inclused in the IPO model is the most meaningfull one and that the relevance of the PTD correction term decreases on increasing the corner size.

**TABLE 3.**

| Normal polarization | Mean square error (dB) | |
|---|---|---|
| $a/\lambda$ = 3.333 | $0° \leq \phi \leq 90°$ | $40° \leq \phi \leq 50°$ |
| PO | -23.69 | -24.76 |
| PO+PTD | -26.01 | -23.19 |
| IPO | -31.23 | -30.32 |

**TABLE 4.**

| Normal polarization | Mean square error (dB) | |
|---|---|---|
| $a/\lambda$ = 5.607 | $0° \leq \phi \leq 90°$ | $40° \leq \phi \leq 50°$ |
| PO | -28.81 | -26.28 |
| PO+PTD | -28.91 | -26.44 |
| IPO | -36.07 | -38.19 |

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